International Journal of Precious Engineering Research and Applications (IJPERA)
www.ijpera.com ISSN: 2456-2734,
Volume 3, Issue 1 (Jan- May) 2018), PP. 37-40

Formation of Complete Bipartite Graphs Using Quaternary Complex Hadamard Matrices

W. V. Nishadi[1], K. D. E. Dhananjaya[1], A. A. I. Perera[1], P. Gunathilake[2]
[1]*(Department of Mathematics, Faculty of Science/ University of Peradeniya, Srilanka)*
[2]*(Department of Computer Science, Faculty of Science/ University of Peradeniya, Srilanka)*
Corresponding Author: W. V. Nishadi

ABSTRACT- A graph G is a complete bipartite graph if its set of vertices could be decomposed into two disjoint sets such that every vertex in first set is joint to all the vertices in second set. In this paper, we proposed an algorithm which can be used to construct complete bipartite graphs using quaternary complex Hadamard matrices. The simplest quaternary complex Hadamard matrix of order 2 is used to obtain those complete bipartite graphs $K_{m,m}$ where $m = 2^n; n \in \mathbb{Z}^+$. This algorithm was tested manually for $m = 2,4,8$. Further, higher order complete bipartite graphs were tested using Java program.

Date of Submission: 30-01-2018 Date of acceptance: 12-02-2018

I. INTRODUCTION

In our previous work [1], we have used normalized Hadamard matrices to construct complete bipartite graphs. In this paper, a construction is generalized using quaternary complex Hadamard matrices.

Let n be even. A **quaternary complex Hadamard** matrix of order n is an $n \times n$ matrix H with entries from $\{\pm 1, \pm i\}$ such that $H(\overline{H})^T = nI_n$ where I_n is the identity matrix of order n [2], [3].

For example, a quaternary complex Hadamard matrix of order *2* is $C = \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}$.

Kronecker product
If **A** is an $m \times n$ matrix and **B** is a $p \times q$ matrix, then the Kronecker product $A \otimes B$ is the $mp \times nq$ block matrix [4].

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \dots & a_{mn}B \end{bmatrix}$$

A graph is said to be a **complete bipartite graph** its vertices can be decomposed into two disjoint sets such that every vertex in first set is joint to all the vertices in second set and it is denoted by $K_{m,n}$ where first set has m vertices and second set has n vertices [5], [6].
In this work, complete graph of $K_{m,m}$ where $m = 2^n$; $n \in \mathbb{Z}^+$ will be constructed.
A graph can be represented using a matrix. In our work, we use **adjacency matrix** which is a $(0,1)$**- matrix**. The graph with n vertices is an $n \times n$ matrix whose (i, j) entry is 1, if i^{th} vertex and j^{th} vertex are connected and 0 otherwise [7].

II. METHODOLOGY

A recursive algorithm is developed using columns of the quaternary complex Hadamard matrices of order 2. Further, Java program is developed to obtain higher order graphs.

2.1 $K_{2,2}$ construction

Consider the quaternary complex Hadamard matrix of order 2 and label its columns (column vectors) as c_1 and c_2.

$$C = \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \end{bmatrix}$$

Multiply each column vector by the transpose of its conjugate and label the resulting block matrices as C_i for $i = 1, 2$.

$$C_1 = c_1\bar{c}_1{}^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$C_2 = c_2\bar{c}_2{}^T = \begin{bmatrix} -i \\ i \end{bmatrix} \cdot \begin{bmatrix} i & -i \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

By using the Kronecker product $C_1 \otimes C_2 = \begin{bmatrix} C_2 & C_2 \\ C_2 & C_2 \end{bmatrix}$, the block matrix of order 4 can be constructed.

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

Replacing -1 by 1 and 1 by 0 the following adjacency matrix of $K_{2,2}$ can be obtained.

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

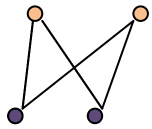

Fig. 01- Complete bipartite graph $K_{2,2}$ of obtained from proposed method

2.2 $K_{4,4}$ Construction

The Kronecker product of C_1 and $C_1 \otimes C_2$ denoted by $C_1 \otimes (C_1 \otimes C_2)$ gives block matrix of order 8 and replacing -1 by 1 and 1 by 0 obtained the adjacency matrix for $K_{4,4}$.

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

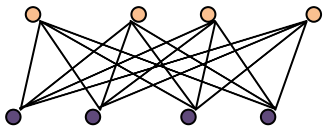

Fig. 02- Complete bipartite graph $K_{4,4}$ of obtained from proposed method

2.3 $K_{8,8}$ Construction

The Kronecker product of C_1 and $C_1 \otimes (C_1 \otimes C_2)$ denoted by $C_1 \otimes (C_1 \otimes (C_1 \otimes C_2))$ gives block matrix of order 16 and replacing -1 by 1 and 1 by 0 obtained the following adjacency matrix for $K_{8,8}$.

$$\begin{bmatrix}
0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0\\
0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0\\
0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0\\
0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0\\
0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0\\
0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0\\
0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0\\
0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1\\
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0
\end{bmatrix}$$

By repeatedly applying the algorithm $K_{m,m}$ where $m = 2^n$; $n \in \mathbb{Z}^+$ can be constructed from the quaternary complex Hadamard matrix of order 2.

III. RESULTS AND DISCUSSION

The following figures are obtained from the above computer program.

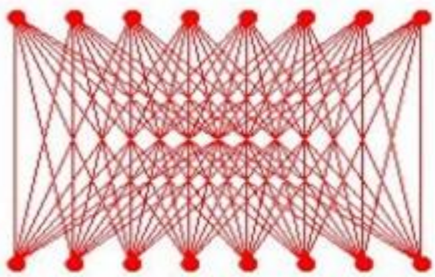

Fig. 03- Complete bipartite graph of $K_{8,8}$ obtained from computer program

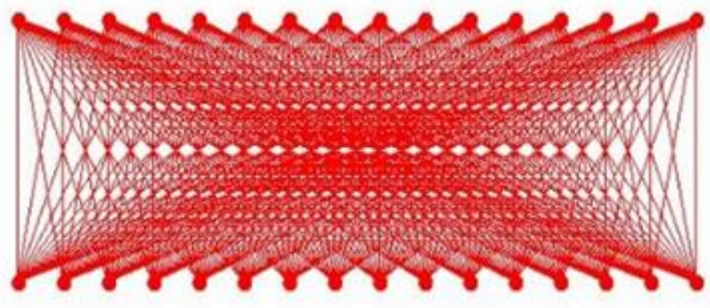

Fig. 04- Complete bipartite graph of $K_{16,16}$ obtained from computer program

Fig. 05- Complete bipartite graph of $K_{64,64}$ obtained from computer program

IV. CONCLUSION

Using the properties of quaternary complex Hadamard matrices an algorithm was developed to construct complete bipartite graphs $K_{m,m}$ where $m = 2^n$ for $n \in \mathbb{Z}^+$. A Computer program has been developed using Java Programming and C+ language for the above construction for higher values of *n*.

Our result is an alternative approach to construct complete bipartite graphs (Sumaiya, 2017) using quaternary complex Hadamard matrix of order 2.

REFERENCES

[1]. M. N. F. Sumaiya, W. V. Nishadi, K. D. E. Dhananjaya, A. A. I. Perera, Generation of Complete Bipartite graphs using Normalized Hadamard Matrices, Elixir International Journal, Elixir Appl. Math. 113, 2017, 49269-49271

[2]. S. S. Again, Hadamard Matrices and Their Applications (LNM 1168, Springer, Berlin. ,1985)

[3]. K. J. Horadam, Hadamard Matrices and Their Application (Princeton University Press, 2007)

[4]. R. A. Horn, C. R. Johnson, Topics in Matrix Analysis, Cambridge University Press, 1991)

[5]. R. Diestel,Graph Theory(In Graduate texts in Mathematics Springer, 2005)

[6]. G. Chartrand, Introductory Graph Theory(New York: Dover, 1985)

[7]. J. Bondy, Graph Theory with application(North Holland, 1976)

W. V. Nishadi "Formation of Complete Bipartite Graphs Using Quaternary Complex Hadamard Matrices“International Journal of Precious Engineering Research and Applications (IJPERA), vol. 03, no. 01, 2018, pp. 37–40.